

Pre-AP Pre- Calculus Sequences and Series Test Review

1. Find the next 3 terms of the recursive sequence

$$\frac{4 + a_{n-1}}{2} \text{ where } a_1 = 8$$

$$a_2 = \frac{4+8}{2} = 6$$

$$a_3 = \frac{4+6}{2} = 5$$

$$a_4 = \frac{4+5}{2} = 4.5$$

2. Write the sum using sigma notation

$$9x + 10x^2 + 11x^3 + 12x^4 + \dots + 98x^{90}$$

$$\sum_{n=1}^{90} (n+8)x^n \text{ or } \sum_{n=9}^{98} nx^{n-8}$$

3. Find the sum.

$$\sum_{k=1}^4 k \cdot 2^k = 1 \cdot 2 + 2 \cdot 4 + 3 \cdot 8 + 4 \cdot 16 = 98$$

4. Find the 18th term of the arithmetic sequence.

$$-t, -t+3, -t+6, -t+9 \dots$$

$$a_{18} = -t + 3(18-1)$$

$$-t + 51$$

5. How many terms are in the arithmetic sequence 3, 8, 13, ..., 73?

$$73 = 3 + 5(n-1)$$

$$n = 15$$

6. A partial sum of an arithmetic sequence is given. Find the sum.

$$3 + 7 + 11 + \dots + 39$$

$$39 = 3 + 4(n-1)$$

$$36 = 4(n-1)$$

$$9 = n-1$$

$$n = 10$$

$$S_{10} = \frac{10}{2} [2 \cdot 3 + 4(10-1)] = \boxed{210}$$

7. Determine whether the sequence is geometric, arithmetic, or neither

6, 24, 96, 384...

geometric
 $r = 4$

8. Determine the n th term of the geometric sequence.

$$x, \frac{x^2}{5}, \frac{x^3}{25}, \frac{x^4}{125}, \dots$$

$$a_n = x \left(\frac{x}{5}\right)^{n-1} \text{ or } \frac{x^n}{5^{n-1}}$$

9. How many terms are in the geometric sequence 5, 20, 80, ..., 20480?

$$20480 = 5(4)^{n-1}$$

$$4096 = 4^{n-1}$$

$$4^6 = 4^{n-1}$$

$$\boxed{n = 7}$$

10. Find the sum.

$$1 + 4 + 16 + \dots + 4096$$

$$4096 = 1(4)^{n-1}$$

$$4^6 = 4^{n-1}$$

$$n = 7$$

$$S_7 = 1 \left(\frac{1-4^7}{1-4}\right)$$

$$= \boxed{5461}$$

11. Find the sum of the infinite geometric series.

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$$

$$r = \frac{1}{3} \quad S = \frac{1}{1-r} = \frac{3}{2} \text{ or } 1.5$$

12. Use the Binomial Theorem to expand the expression $(3-x)^5$.

$$\binom{5}{0}3^5 + \binom{5}{1}3^4(-x) + \binom{5}{2}3^3(-x)^2 + \binom{5}{3}3^2(-x)^3 + \binom{5}{4}3(-x)^4 + \binom{5}{5}(-x)^5$$

$$1 \cdot 243 + -5 \cdot 81x + 10 \cdot 27x^2 - 10 \cdot 9x^3 + 5 \cdot 3x^4 - 1 \cdot 1 \cdot x^5$$

$$\boxed{243 - 405x + 270x^2 - 90x^3 + 15x^4 - x^5}$$

13. Find the 4th term in the expansion of $(x + 2y)^{15}$.

$$\frac{15!}{3! \cdot 12!} = \frac{15 \cdot 14 \cdot 13}{3 \cdot 2} = 455$$

$$\binom{15}{3} x^{12} (2y)^3 = 455 x^{12} \cdot 8y^3 = 3640 x^{12} y^3$$

14. Find the middle term in the expansion of $(x^4 + 1)^{20}$.

$$\frac{20!}{10! \cdot 10!} = 184756$$

$$\binom{20}{10} (x^4)^{10} (1)^{10} = 184756 x^{40}$$

15. Find the term containing x^6 in the expansion of $(x + 2y)^{10}$.

$$\frac{10!}{4! \cdot 6!} = 210$$

$$\binom{10}{4} x^6 (2y)^4 = 210 x^6 \cdot 16y^4 = 3360 x^6 y^4$$

16. Find the sum.

$$\sum_{k=2}^6 2^{k-2} = 31$$

$$2^0 + 2^1 + 2^2 + 2^3 + 2^4$$

17. The 12th term of an arithmetic sequence is 34, and the fifth term is 20. Find the 20th term.

$$a_{20} = 50$$

12 → 5 7 terms
20 → 34 14 terms
 $d = 14/7 = 2$
 $34 = a + 2(12-1)$
 $a = 12$ $a_{20} = 12 + 2(20-1)$

18. A partial sum of an arithmetic sequence is given. Find the sum.

$$\sum_{n=0}^{20} (1 - 7n)$$

$$S = -1449$$

19. An arithmetic sequence has first term $a_1 = 7$ and fourth term $a_4 = 22$. How many terms of this sequence must be added to get 3,402?

$$d = 15/3 = 5$$

$$n = 36$$

$$3402 = \frac{n}{2} [2 \cdot 7 + 5(n-1)]$$

y_1 y_2
find intersection

20. Find the sum of the infinite geometric series.

$$4 + \frac{4}{3} + \frac{4}{9} + \frac{4}{27} + \dots$$

$$r = \frac{1}{3}$$

$$S = \frac{4}{1 - 1/3} = 6$$

21. Write the sum using sigma notation.

$$4 + 8 + 12 + 16 + 20 + 24 + 28 + 32 + 36 + 40$$

$$\sum_{n=1}^{10} 4 + 4(n-1) \text{ or } \sum_{n=1}^{10} 4n \text{ or } \sum_{n=0}^9 4n + 4$$

22. Determine whether the sequence is arithmetic, geometric, or neither.

5, 11, 13, 23, ...

neither

23. Find the sum of the infinite geometric series.

$$1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$$

$$S = \frac{1}{1 - 1/3} = 3/2 \text{ or } 1.5$$

24. Find the sum of the infinite geometric series.

$$\frac{1}{16} + \frac{1}{8} + \frac{1}{4} + \dots$$

$$r = 2$$

diverges