

ALGEBRA
II

The University of the State of New York
REGENTS HIGH SCHOOL EXAMINATION

ALGEBRA II

Wednesday, June 21, 2023 — 9:15 a.m. to 12:15 p.m., only

Student Name Mr. Sibol

School Name JMAP, Inc

The possession or use of any communications device is strictly prohibited when taking this examination. If you have or use any communications device, no matter how briefly, your examination will be invalidated and no score will be calculated for you.

Print your name and the name of your school on the lines above.

A separate answer sheet for **Part I** has been provided to you. Follow the instructions from the proctor for completing the student information on your answer sheet.

This examination has four parts, with a total of 37 questions. You must answer all questions in this examination. Record your answers to the Part I multiple-choice questions on the separate answer sheet. Write your answers to the questions in **Parts II, III, and IV** directly in this booklet. All work should be written in pen, except for graphs and drawings, which should be done in pencil. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale.

The formulas that you may need to answer some questions in this examination are found at the end of the examination. This sheet is perforated so you may remove it from this booklet.

Scrap paper is not permitted for any part of this examination, but you may use the blank spaces in this booklet as scrap paper. A perforated sheet of scrap graph paper is provided at the end of this booklet for any question for which graphing may be helpful but is not required. You may remove this sheet from this booklet. Any work done on this sheet of scrap graph paper will not be scored.

When you have completed the examination, you must sign the statement printed at the end of the answer sheet, indicating that you had no unlawful knowledge of the questions or answers prior to the examination and that you have neither given nor received assistance in answering any of the questions during the examination. Your answer sheet cannot be accepted if you fail to sign this declaration.

Notice ...

A graphing calculator and a straightedge (ruler) must be available for you to use while taking this examination.

DO NOT OPEN THIS EXAMINATION BOOKLET UNTIL THE SIGNAL IS GIVEN.

Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 The population of Austin, Texas from 1850 to 2010 is summarized in the table below.

Year	1850	1870	1890	1910	1930	1950	1970	1990	2010
Population	629	4428	14,575	29,860	53,120	132,459	251,808	494,290	790,390

Over which period of time was the average rate of change in population the greatest?

(1) 1850 to 1910

(3) 1950 to 1970

(2) 1990 to 2010

(4) 1890 to 1970

(3) $\frac{251808 - 132459}{1970 - 1950} \approx 5967$

(1) $\frac{29860 - 629}{1910 - 1850} \approx 487$

(2) $\frac{790390 - 494290}{2010 - 1990} \approx 14805$

(4) $\frac{251808 - 14575}{1970 - 1890} \approx 2965$

2 Which expression is *not* equivalent to $36x^6 - 25y^4$?

(1) $6^2(x^3)^2 - 5^2(y^2)^2$

(3) $(6x^6 - 5y^4)(6x^6 + 5y^4)$

(2) $(6x^3 - 5y^2)(6x^3 + 5y^2)$

(4) $(3 \cdot 2x^3 - 5y^2)(3 \cdot 2x^3 + 5y^2)$

3 What are the zeros of $s(x) = x^4 - 9x^2 + 3x^3 - 27x - 10x^2 + 90$?

(1) $\{-3, -2, 5\}$

(3) $\{-3, -2, 3, 5\}$

(2) $\{-2, 3, 5\}$

(4) $\{-5, -3, 2, 3\}$

$s(x) = x^4 + 3x^3 - 19x^2 - 27x + 90$

Use this space for computations.

4 If θ is an angle in standard position whose terminal side passes through the point $(-2, -3)$, what is the numerical value of $\tan \theta$?

(1) $\frac{2}{3}$

(3) $-\frac{2}{\sqrt{13}}$

(2) $\frac{3}{2}$

(4) $-\frac{3}{\sqrt{13}}$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{-3}{\sqrt{13}}}{\frac{-2}{\sqrt{13}}} = \frac{-3}{-2} = \frac{3}{2}$$

5 The average monthly temperature, $T(m)$, in degrees Fahrenheit, over

a 12 month period, can be modeled by $T(m) = -23 \cos\left(\frac{\pi}{6}m\right) + 56$,

where m is in months. What is the range of temperatures, in degrees Fahrenheit, of this function?

(1) $[-23, 23]$

(3) $[-23, 56]$

(2) $[33, 79]$

(4) $[-79, 33]$

$$\begin{aligned} -23(-1) + 56 &= 79 \\ -23(1) + 56 &= 33 \end{aligned}$$

6 Which expression is an equivalent form of $a\sqrt[5]{a^4}$?

(1) a

(3) $a^{\frac{9}{4}}$

(2) $a^{\frac{9}{5}}$

(4) $a^{\frac{1}{5}}$

$$a^{\frac{5}{5}} \cdot a^{\frac{4}{5}} = a^{\frac{9}{5}}$$

7 The expression $3i(ai - 6i^2)$ is equivalent to

(1) $3a + 18i$

(3) $-3a + 18i$

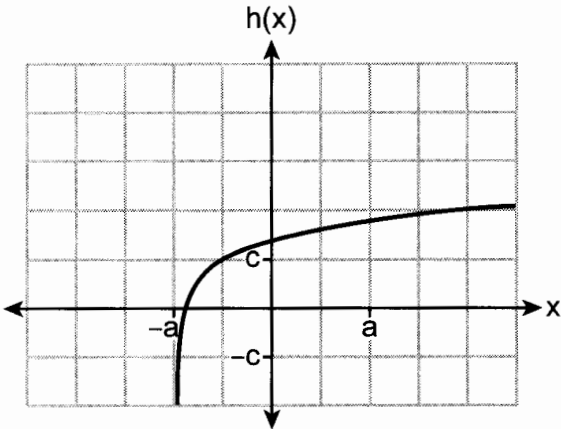
(2) $3a - 18i$

(4) $-3a - 18i$

$$\begin{aligned} &3ai^2 - 18i^3 \\ &3a(-1) - 18(-i) \\ &-3a + 18i \end{aligned}$$

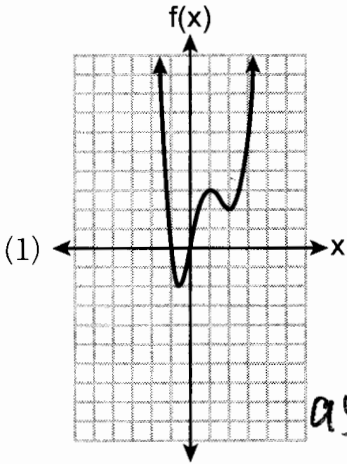
Use this space for computations.

8 Which equation best represents the graph below?



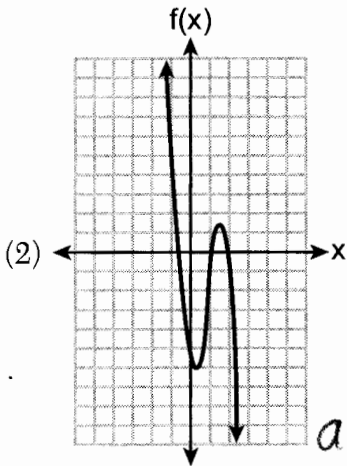
- (1) $h(x) = \log(x + a) + c$ (3) $h(x) = \log(x + a) - c$
 (2) $h(x) = \log(x - a) + c$ (4) $h(x) = \log(x - a) - c$

9 Which function has the characteristic as $x \rightarrow -\infty, f(x) \rightarrow -\infty$?



(3) $f(x) = 5(4)^{-x}$ as $x \rightarrow -\infty, f(x) \rightarrow \infty$

as $x \rightarrow -\infty, f(x) \rightarrow +\infty$



(4) $f(x) = -\log_5(-x)$

as $x \rightarrow -\infty, f(x) \rightarrow$

Use this space for computations.

10 The expression $(x^2 + 3)^2 - 2(x^2 + 3) - 24$ is equivalent to

- (1) $(x^2 + 9)(x^2 - 1)$ (3) $x^4 - 2x^2 - 21$
 (2) $(x^2 - 3)(x^2 + 7)$ (4) $x^4 + 4x^2 - 9$

$$u^2 - 2u - 24$$

$$(u-6)(u+4)$$

$$(x^2+3-6)(x^2+3+4)$$

11 What is the solution for the system of equations below?

$$2x - y = -2$$

$$\begin{cases} x + y + z = 2 \\ x - 2y - z = -4 \\ x - 9y + z = -18 \end{cases}$$

$$\begin{array}{r} 2x - y = -2 \\ 2x - 11y = -22 \\ \hline 10y = 20 \\ y = 2 \end{array}$$

$$\begin{array}{r} x + 2 + z = 2 \\ x - 2(2) - z = -4 \\ \hline x + z = 0 \\ x - z = 0 \\ \hline 2x = 0 \\ x = 0 \\ z = 0 \end{array}$$

- (1) $(-2, 2, 2)$ (3) $(0, 2, 0)$
 (2) $(-2, -2, 6)$ (4) $(0, 2, 4)$

12 The roots of the equation $x^2 - 4x = -13$ are

- (1) $2 \pm 3i$ (3) $2 \pm \sqrt{17}$
 (2) $2 \pm 6i$ (4) $2 \pm i\sqrt{13}$

$$x^2 - 4x + 4 = -13 + 4$$

$$(x-2)^2 = -9$$

$$x-2 = \pm 3i$$

$$x = 2 \pm 3i$$

13 Which expression is equivalent to $\frac{2x^3 + 2x - 7}{2x + 4}$?

- (1) $x^2 - 2x + 5 - \frac{27}{2x + 4}$ (3) $x^2 + 2x + 5 + \frac{13}{2x + 4}$
 (2) $x^2 - 1 - \frac{3}{2x + 4}$ (4) $x^2 + 2x - 3 + \frac{5}{2x + 4}$

$$\begin{array}{r} x^2 - 2x + 5 \\ 2x+4 \overline{) 2x^3 + 0x^2 + 2x - 7} \\ \underline{2x^3 + 4x^2} \\ -4x^2 + 2x \\ \underline{-4x^2 - 8x} \\ 10x - 7 \\ \underline{10x + 20} \\ -27 \end{array}$$

Use this space for computations.

- 14 A popular celebrity tracks the number of people, in thousands, who have followed her on social media since January 1, 2015. A summary of the data she recorded is shown in the table below:

Number of Months Since January 2015	2	11	16	20	27	35	47	50	52
Number of Social Media Followers (thousands)	3.1	7.5	29.7	49.7	200.3	680.3	5200.3	8109.3	12,107.1

The celebrity uses an exponential regression equation to model the data. According to the model, about how many followers did she have on June 1, 2018?

- (1) 13,000,000
(2) 5,420,000

- (3) 1,850,000
(4) 790,000

$$y = 1.77(1.18)^x$$

$$y(41) \approx 1,850,950$$

- 15 Luminescence is the emission of light that is not caused by heat. A luminescent substance decays according to the function below.

$$I = I_0 e^{3\left(-\frac{t}{0.6}\right)}$$

This function can be best approximated by

- (1) $I = I_0 e^{\left(-\frac{t}{0.18}\right)}$
(2) $I = I_0 e^{5t}$

- (3) $I = I_0(0.0067)^t$
(4) $I = I_0(0.0497)^{0.6t}$

$$e^{-\frac{3}{0.6}} \approx 0.006738$$

- 16 The heights of the students at Central High School can be modeled by a normal distribution with a mean of 68.1 and a standard deviation of 3.4 inches. According to this model, approximately what percent of the students would have a height less than 60 inches or greater than 75 inches?

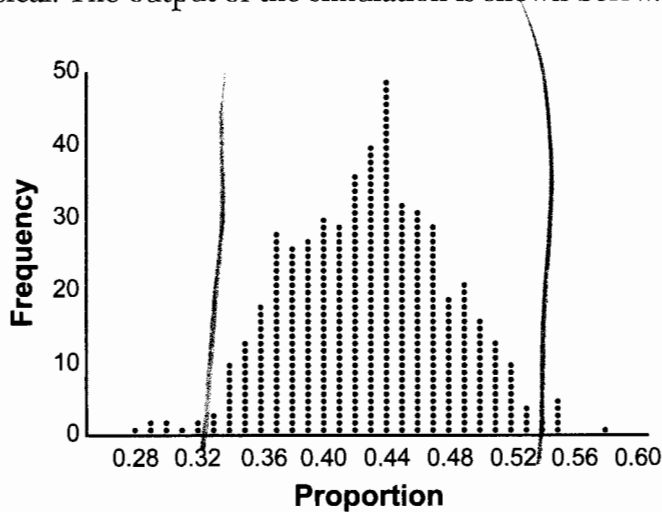
- (1) 0.86%
(2) 1.26%

- (3) 2.12%
(4) 2.98%

$$\begin{array}{r} .008601 < 60 \\ .021208 > 75 \\ \hline .029809 \end{array}$$

Use this space for
computations.

- 17 Marissa and Sydney are trying to determine if there is enough interest in their school to put on a senior musical. They randomly surveyed 100 members of the senior class and 43% of them said they would be interested in being in a senior musical. Marissa and Sydney then conducted a simulation of 500 more surveys, each of 100 seniors, assuming that 43% of the senior class would be interested in being in the musical. The output of the simulation is shown below.



The standard deviation of the simulation is closest to

- (1) 0.02 (3) 0.09
(2) 0.05 (4) 0.43

$.43 \pm 2(0.05)$
contains about 95%
of the data

- 18 For $f(x) = \cos x$, which statement is true?

- (1) $2f(x)$ and $f(2x)$ are even functions.
(2) $f(2x)$ and $f(x) + 2$ are odd functions.
(3) $2f(x)$ and $f\left(x + \frac{\pi}{2}\right)$ are odd functions.
(4) $f(x) + 2$ is an odd function and $f\left(x + \frac{\pi}{2}\right)$ is an even function.

Use this space for computations.

19 The solution set of $\frac{x+3}{x-5} + \frac{6}{x+2} = \frac{6+10x}{(x-5)(x+2)}$ is

- (1) $\{-6\}$ (3) $\{-6, 5\}$
 (2) $\{5\}$ (4) $\{-5, 6\}$

$$\frac{(x+3)(x+2) + 6(x-5)}{(x-5)(x+2)} = \frac{10x+6}{(x-5)(x+2)}$$

20 Given x and y are positive, which expressions are equivalent to $\frac{x^3}{y}$? $x^2 + 5x + 6 + 6x - 30 = 10x + 6$

$$\frac{x^3}{y} = \frac{x^6 y^{\frac{1}{4}}}{x^3 y^3}$$

- I. $\left(\frac{y}{x^3}\right)^{-1} = \frac{x^3}{y}$
 II. $\sqrt[3]{x^9}(y^{-1}) = \frac{x^3}{y}$
 III. $\frac{x^6 \sqrt[4]{y^8}}{x^3 y^3} = \frac{x^3}{y}$

$$x^2 + 11x - 24 = 10x + 6$$

$$x^2 + x - 30 = 0$$

$$(x+6)(x-5) = 0$$

$$x = -6, 5$$

- (1) I and II; only (2) I and III; only
 (3) II and III; only (4) I, II, and III

21 Given the inverse function $f^{-1}(x) = \frac{2}{3}x + \frac{1}{6}$, which function represents $f(x)$?

- (1) $f(x) = -\frac{2}{3}x + \frac{1}{6}$ (3) $f(x) = \frac{3}{2}x - \frac{1}{4}$
 (2) $f(x) = -\frac{3}{2}x + \frac{1}{4}$ (4) $f(x) = \frac{3}{2}x - \frac{1}{6}$

$$x = \frac{2x}{3} + \frac{1}{6}$$

$$6x = 4x + 1$$

$$\frac{6x-1}{4} = \frac{4x}{4}$$

$$\frac{2}{3}x - \frac{1}{4} = y$$

Use this space for computations.

22 How many equations below are identities?

- $x^2 + y^2 = (x^2 - y^2) + (2xy)^2$
- $x^3 + y^3 = (x - y) + (x^2 - xy + y^2)$
- $x^4 + y^4 = (x - y)(x - y)(x^2 + y^2)$

(1) 1

(2) 2

(3) 3

(4) 0

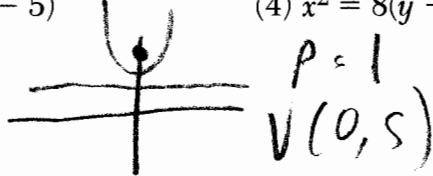
23 If the focus of a parabola is $(0, 6)$ and the directrix is $y = 4$, what is an equation for the parabola?

(1) $y^2 = 4(x - 5)$

(3) $y^2 = 8(x - 5)$

(2) $x^2 = 4(y - 5)$

(4) $x^2 = 8(y - 6)$



$$y - 5 = \frac{1}{4(1)} (x - 0)^2$$
$$4(x - 0)^2 = x^2$$

24 John and Margaret deposit \$500 into a savings account for their son on his first birthday. They continue to make a deposit of \$500 on the child's birthday, with the last deposit being made on the child's 21st birthday. If the account pays 4% annual interest, which equation represents the amount of money in the account after the last deposit is made?

(1) $S_{21} = 500(1.04)^{21}$

(3) $S_{21} = 500(1.04)^{20} + 500$

(2) $S_{21} = \frac{500(1 - 1.04^{21})}{1 - 1.04}$

(4) $S_{21} = \frac{500(1 - 0.04^{21})}{1 - 1.04}$

Part II

Answer all 8 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

- 25 The business office of a local college wishes to determine the methods of payment that will be used by students when buying books at the beginning of a semester. Explain how the office can gather an appropriate sample that minimizes bias.

Pick random names from a list of all students & ask each student his method.

26 Determine the solution of $\sqrt{3x + 7} = x - 1$ algebraically.

$$3x + 7 = x^2 - 2x + 1$$

$$0 = x^2 - 5x - 6$$

$$0 = (x - 6)(x + 1)$$

$$x = 6, \text{ } \cancel{x = -1}$$

- 27 The population of bacteria, $P(t)$, in hundreds, after t hours can be modeled by the function $P(t) = 37e^{0.0532t}$. Determine whether the population is increasing or decreasing over time. Explain your reasoning.

Increasing, because $e^{0.0532} > 1$

- 28 The polynomial function $g(x) = x^3 + ax^2 - 5x + 6$ has a factor of $(x - 3)$. Determine the value of a .

$$g(3) = 0$$

$$0 = 3^3 + 3^2a - 5(3) + 6$$

$$0 = 27 + 9a - 15 + 6$$

$$-18 = 9a$$

$$-2 = a$$

29 Write a recursive formula for the sequence 189, 63, 21, 7,

$$a_1 = 189$$
$$a_n = \frac{1}{3} a_{n-1}$$

30 Solve algebraically for x to the nearest thousandth:

$$2e^{0.49x} = 15$$

$$\ln 2e^{0.49x} = \ln 15$$

$$\frac{0.49x}{0.49} = \frac{\ln 15}{0.49}$$

$$x \approx 4.112$$

31 For all values of x for which the expression is defined, write the expression below in simplest form.

$$\frac{2x^3 + x^2 - 18x - 9}{3x - x^2}$$

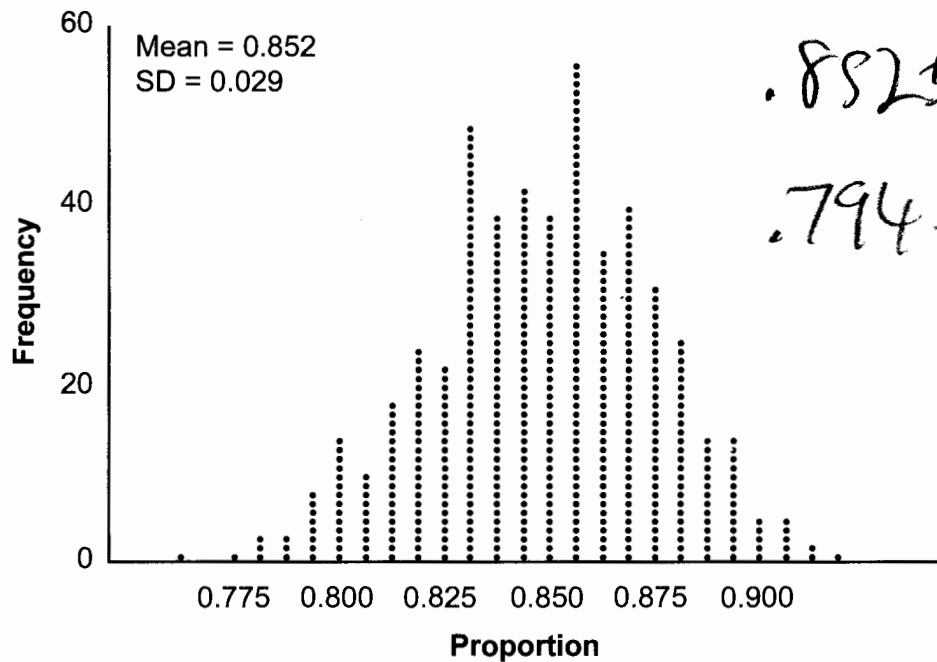
$$\frac{x^2(2x+1) - 9(2x+1)}{x(3-x)}$$

$$\frac{(x^2-9)(2x+1)}{x(3-x)}$$

$$\frac{(x+3)\cancel{(x-3)}(2x+1)}{-x\cancel{(x-3)}}$$

$$\frac{(x+3)(2x+1)}{-x}$$

- 32 An app design company believes that the proportion of high school students who have purchased apps on their smartphones in the past 3 months is 0.85. A simulation of 500 samples of 150 students was run based on this proportion and the results are shown below.



$$.852 \pm 2(.029)$$
$$.794 - .91$$

Suppose a sample of 150 students from your high school showed that 88% of students had purchased apps on their smartphones in the past 3 months. Based on the simulation, would the results from your high school give the app design company reason to believe their assumption is *incorrect*? Explain.

No, because .88 falls within the confidence interval

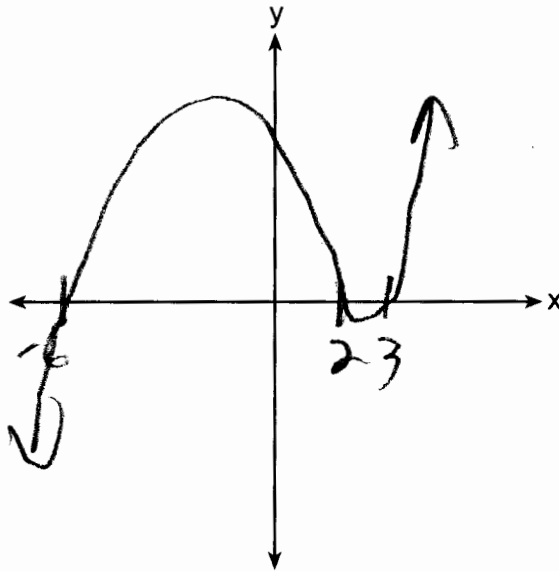
Part III

Answer all 4 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [16]

- 33 Patricia creates a cubic polynomial function, $p(x)$, with a leading coefficient of 1. The zeros of the function are 2, 3, and -6 . Write an equation for $p(x)$.

$$p(x) = (x-2)(x-3)(x+6)$$

Sketch $y = p(x)$ on the set of axes below.



- 34 A public radio station held a fund-raiser. The table below summarizes the donor category and method of donation.

		Donor Category	
		Supporter	Patron
Method of Donation	Phone calls	400	672
	Online	1200	2016

To the *nearest thousandth*, find the probability that a randomly selected donor was categorized as a supporter, given that the donation was made online.

$$\frac{1200}{1200 + 2016} \approx .373$$

Do these data indicate that being a supporter is independent of donating online? Justify your answer.

$$\text{Yes, } \frac{1600}{4288} \approx .373.$$

Independent, since equal

35 Algebraically solve the system:

$$(x - 2)^2 + (y - 3)^2 = 20$$

$$y = -2x + 7$$

$$(x-2)^2 + (-2x+7-3)^2 = 20$$
$$x^2 - 4x + 4 + 4x^2 - 16x + 16 = 20$$

$$5x^2 - 20x = 0$$

$$5x(x-4) = 0$$

$$x = 0, 4$$

$$y = -2(0) + 7$$
$$= 7$$

$$(0, 7)$$

$$y = -2(4) + 7$$
$$y = -1$$

$$(4, -1)$$

36 On a certain tropical island, there are currently 500 palm trees and 200 flamingos. Suppose the palm tree population is decreasing at an annual rate of 3% per year and the flamingo population is growing at a continuous rate of 2% per year.

Write two functions, $P(x)$ and $F(x)$, that represent the number of palm trees and flamingos on this island, respectively, x years from now.

$$P(x) = 500(.97)^x$$

$$F(x) = 200e^{.02x}$$

State the solution to the equation $P(x) = F(x)$, rounded to the *nearest year*. Interpret the meaning of this value within the given context.

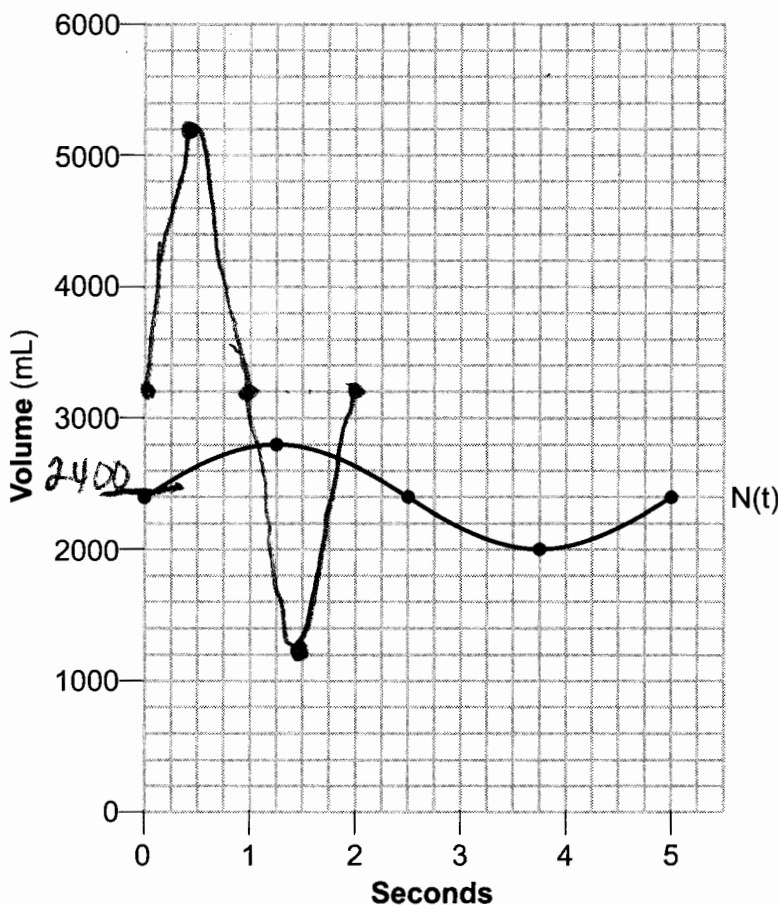
18

The number of palm trees & Flamingos will be equal in 18 years

Part IV

Answer the question in this part. A correct answer will receive 6 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided to determine your answer. Note that diagrams are not necessarily drawn to scale. A correct numerical answer with no work shown will receive only 1 credit. The answer should be written in pen. [6]

37 The volume of air in an average lung during breathing can be modeled by the graph below.



$$b = \frac{2\pi}{5}$$

Using the graph, write an equation for $N(t)$, in the form $N(t) = A \sin(Bt) + C$.

$$N(t) = 400 \sin \frac{2\pi}{5} t + 2400$$

Question 37 is continued on the next page.

Question 37 continued

That same lung, when engaged in exercise, has a volume that can be modeled by $E(t) = 2000 \sin(\pi t) + 3200$, where $E(t)$ is volume in mL and t is time in seconds.

Graph *at least one* cycle of $E(t)$ on the same grid as $N(t)$.

$$\text{period} = \frac{2\pi}{\pi} = 2$$

How many times during the 5-second interval will $N(t) = E(t)$?

4